Roll Number		

Code Number 41/2

INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION
MATHEMATICS

CLASS: XII

Sub. Code: 041

Time Allotted: 3 Hrs

Max. Marks: 100

14.09.2017

General Instructions:

(i) **All** questions are compulsory..

(ii) This question paper contains **29** questions.

(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.

(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.

(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.

(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

SECTION - A (4 x 1 = 6)

1. Evaluate $\int e^{2x+3}dx$

2. Let f(x)=$\begin{cases} 4x-5 : x \geq 2 \\ 1-x : x < 2 \end{cases}$ and

g(x) =$\begin{cases} 3x+7 : x \leq 3 \\ 7 : x > 3 \end{cases}$ be a function from R to R . Find (gof)(9)

3. Operate $C_2 \to C_2 - 2C_1$

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$$

4. If $\tan^{-1}(\cot\theta) = 2\theta$, then θ = --------

SECTION - B (8 x 2 = 16)

5. f(x) =x^2 - x + 1 , find fof and evaluate fof (1).

6. Find the values of the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) + 2\cot^{-1}(-1)$$

7. Write the following in the simplest form:

$$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), |x| > 1$$

8. If area of the triangle is 35sq.units with vertices (2,-6) ,(5,4) , (k,4),find the value of k.

9. Find the values of k, if the following function

$$f(x) = \begin{cases} kx+1, if\ x \le 5 \\ 3x-5, x>5 \end{cases} \text{ is continuous at } x = 5$$

10. Find $\frac{dy}{dx}$, $if\ y = \tan^{-1}(\frac{cosx-sinx}{cosx+sinx})$.

11. A car starts from a point P at time t = 0 seconds and stops at point Q.The distance x, in meters,covered by it in t seconds is given by $x = t^2(2 - \frac{t}{3})$.Find the time taken by it to reach Q. and also find distance between P and Q.

12. Find the points on the curve $y = 2x^2-6x-4$ at which the tangent is parallel to the x-axis.

SECTION - C **(11x4=44)**

13. Let f: N →N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$$ for all $n \in$ N, State whether the function f is bijective.

14. Solve : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

(OR)

Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$

15. Prove that $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4}\right)$

16. Using properties prove $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

17. Find the value of a and b , if $f(x) = \begin{cases} x + a\sqrt{2}\ sinx, 0 \le x \le \frac{\pi}{4} \\ 2xcotx + b, \frac{\pi}{4} \le x < \frac{\pi}{2} \\ acos2x - bsinx, \frac{\pi}{2} \le x < \pi \end{cases}$ is continuous on $[0, \pi]$

18. x = a(cost +t sin t) , y = a(sin t – t cos t) . Find $\frac{d^2y}{dx^2}$.

(OR)

$y = \sin^{-1}(\frac{2^{x+1}}{1+4^x})$ .Find $\frac{dy}{dx}$.

19. Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point, where it cuts x-axis.

20. Find the intervals in which the functions given below are a)strictly decreasing b) strictly Increasing:

$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

21. Evaluate : $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$

22. Evaluate : $\int \frac{1}{1+\tan x} dx$

23. Evaluate : $\int \frac{e^{2x}-1}{e^{2x}+1} dx$ (OR) $\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} dx$

SECTION - D

(6x6=36)

24. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

(OR)

A window has the shape of a rectangle surmounted by an equilateral triangle . If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

25. Find the absolute maximum and absolute minimum of the following.
$f(x) = 2x^3 - 15x^2 + 36x + 1$ on [1 , 5]

26. If $\cos\frac{x}{2}.\cos\frac{x}{4}.\cos\frac{x}{8}\ldots\ldots = \frac{\sin x}{x}$,

Then prove that $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{4} + \text{------} = \operatorname{cosec}^2 x - \frac{1}{x^2}$

27. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3 , 2 and 1 students respectively with a total award money Rs 2200. School Q wants to spend Rs 3100 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs 1200, using matrices ,find the award money for each value.

Apart from these three values, suggest one more value which should be considered for award.

28. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11I = O$. Hence find A^{-1} & A^4

OR

Obtain the inverse of the matrix using elementary transformations: $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

29. Consider the operation * on Q-{-1} defined by a*b=a+ b + ab. a, b ε Q - {-1}.Prove that * is binary. 2) Is * commutative? 3) Is * associative 4) does the identity for * exist? If yes, find the identity .5) Are elements of Q invertible ? If so find the inverse of a rational numbers other than -1.

(OR)

Show that the relation R in the set A={1,2,3,4,5} given by $R=\{(a,b): |a-b| is\ even\}$ is an equivalence relation. Show that all the Elements of {1,3,5} *are* related to each other & all the elements of {2,4} are related to each other. But No elements of{1,3,5} is related to any element of {2,4}.

End of the Question Paper